PAPER ON MATHEMATICS COURSES AND INCENTIVES

This is an update of a discussion paper on Mathematics courses and incentives published in March 2008. Since then the four public WA universities have endorsed the use of incentives in the Mathematics courses from and including 2010 Year 12 as explained in this document.

RANKING FOR UNIVERSITY ADMISSION

Entry into most courses at the four public WA universities is competitive, that is for a significant number of university courses there are usually more applicants wishing to do a particular course than places available. Because of this, applicants have to be ranked on the basis of their likely success at university. The best measure of future academic achievement is past success in academic studies. For school leavers, this is their performance in the WACE / TEE exams and their school assessments. The Tertiary Entrance Rank (TER) is a ranking of students’ academic achievement taking into account their exam and school performance in their best four WACE courses/TEE subjects.

To be fair to all students regardless of the courses/subjects studied and school attended, it is necessary to apply statistical processes such as standardisation, moderation and scaling before determining a student’s TER.

INTRODUCTION OF STAGE 3 INCREMENT

In January 2007 the Minister for Education announced that there would be separate stage 2 and stage 3 examinations in all WACE courses. These examinations are to be held for the first time in 2009.

The TISC Management Committee endorsed the recommendation by Dr Partis in his paper, Report on Scaling stage 2 and stage 3 results, that an increment of 15 be applied to the combined unscaled stage 3 course results, in recognition that stage 3 units are significantly more difficult than stage 2 units and as an incentive for students to undertake the higher level units of the course, if they are capable of doing so. The increment will be reviewed in 2010 and 2011 once there is actual data available.

Appendix 1 is a flow diagram of how the raw exam and school assessments for stage 2 and stage 3 course results will be processed from 2009. Note that the standardisation and moderation processes (the same as for the existing TEE subjects) are applied to the stage 2 and stage 3 results separately before applying the increment to stage 3 results and producing a merged course combined mark distribution of results. The merged combined mark distributions for all courses are then scaled.

ALL WACE COURSES EXCEPT MATHEMATICS

For all courses except Mathematics, there will be two exams from 2009: one at stage 2 and the other at stage 3. To be fair to students undertaking the more difficult stage 3 units an increment of 15 added to the stage 3 combined marks will be used, as explained earlier.

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MATHEMATICS COURSES

In the case of Mathematics there will be two courses introduced in Year 11 in 2009, with six exams in 2010:

Mathematics (MAT) with two stage 2 (2A/2B; 2C/2D) and two stage 3 (3A/3B; 3C/3D) exams

Mathematics: Specialist (MAS) with two stage 3 (3A/3B; 3C/3D) exams

Appendix 2 is an extract from a Curriculum Council document showing the unit pathways for MAT and MAS courses.

Mathematics (MAT)

This course will be undertaken by the majority of students in Senior School wanting to do some mathematics, with units covering the ability levels and content of the existing Year 11 Vocational Mathematics through to TEE Applicable Mathematics subjects in a similar way to the English course which now offers units covering ability levels from Vocational English through to TEE English.

The Mathematics Advisory Group has indicated that the content of the units from 2A/2B through to 3C/3D increases in difficulty significantly between each pair of units.

The four mathematics exams will cover the following stage 2 and stage 3 units:

Low stage 2 Units 2A and 2B
High stage 2 Units 2C and 2D (Average Discrete Mathematics standard)
Low stage 3 Units 3A and 3B (Good Discrete Mathematics standard)
High stage 3 Units 3C and 3D (Applicable Mathematics standard)

Mathematics: Specialist (MAS)

This course has been designed for the very able mathematics students interested in studying Mathematics and Engineering courses at university level.

The Mathematics Advisory Group has indicated that the content of the 3C/3D units is significantly more difficult compared to the content of the 3A/3B units.

There will be two stage 3 exams in Mathematics: Specialist covering:

Low stage 3 Units 3A and 3B (Good Introductory Geometry & Trigonometry standard)
High stage 3 Units 3C and 3D (Calculus standard)

PROBLEM

Because of the range and depth of specific content covered by the Mathematics course, it is necessary to have four exams covering the varying stages of difficulty. No other course has this requirement.

Given the unit structure in Mathematics, how do you encourage students to attempt the highest level of mathematics they are capable of, as well as being consistent with the decision to use an increment of 15 between stage 2 and stage 3 in all non-mathematics courses?
ANALYSIS

A/Prof Ken Harrison from Murdoch University undertook extensive modelling of a number of possible participation distributions in mathematics unit pairs 2A/B, 2C/D, 3A/B and 3C/D to determine what increment between the unit pairs would produce the net effect of an increment of 15 between the combined 2A/B, 2C/D (stage 2) and combined 3A/B, 3C/D (stage 3). He used increments of 7, 10 and 12. See Appendix 3 for A/Prof Harrison’s analysis.

The analysis showed that, regardless of the percentage distributions between the mathematics unit pairs and variations in the percentage distribution between stage 2 and stage 3 in a typical non – mathematics course (mean 58 and standard deviation 14) used for comparison, an increment of 10 between the mathematics unit pairs results in a difference very close to 15 between the stage 2 and stage 3 mathematics distributions.

For details, see figures 1 to 6 in Appendix 3, which show that the use of an increment of 10 between the mathematics pairs results in the stage 2 and stage 3 mathematics distributions matching very closely the non – mathematics distributions of stage 2 and stage 3 where the increment is 15.

SOLUTION

1. Within the Mathematics course unit pairs will be separated by applying an increment of 10 to the combined marks as follows:
   - Combined marks for Units 2A /2B no increment
   - Combined marks for Units 2C /2D + 10
   - Combined marks for Units 3A /3B + 20
   - Combined marks for Units 3C /3D + 30

This provides the incentive to encourage students to attempt the highest level of mathematics they are capable of, as well as recognising there are four stages of difficulty within the Mathematics course that will be examined, compared to two stages in the other courses. The use of an increment of 10 between unit pairs is also consistent with the use of the increment of 15 between stage 2 and stage 3 in other courses.

Appendix 4 is a flow diagram of how the raw exam and school assessments results for the mathematics unit pairs would be processed from 2010. Note that the standardisation and moderation processes (the same as for the existing TEE subjects) are applied to the results of the unit pairs separately before producing a merged course combined mark distribution of results before scaling.

2. Within the Mathematics: Specialist course an increment of 10 will be applied to the combined marks of the 3C/3D results relative to the 3A/3B combined marks.

This is consistent with 1 above, where an increment of 10 is to be used between unit pairs within a stage. Appendix 5 is the flow diagram of how the raw exam and school assessments results for the Mathematics: Specialist unit pairs will be processed from 2010.

3. In 2011, the mathematics increments as well as the increment for the other courses will be reviewed. Changes, if any, recommended by this review are unlikely to be implemented before 2013 Year 12.
FAQs

Will students undertaking the Mathematics units 3C/3D be advantaged because they will receive an increment of 30 compared to an increment of 15 for stage 3 in other courses?

The answer is no. The increments provide the means of helping to rank the students within the Mathematics course taking into account the relative difficulties of the four unit pairs.

The first stage of the Average Marks Scaling process is to convert all merged combined marks distributions to normal distributions with means of 0 and standard deviations of 1. That is, the Mathematics distribution with its greater range of merged combined marks will be converted (shrunk) to the same distribution used for all the other courses before scaling.

Is 10 the appropriate increment value to be used between each of the Mathematics pairs of units?

It is important to have an incentive of a reasonable value to encourage mathematics students to undertake the more difficult mathematics units, if they are capable of doing so.

At the same time, it is important that the increment does not give poor performance in units 3C/3D an unwarranted boost that might result in an inappropriately high mark compared with good performance at a lower level. For this reason an increment of 15 between each pair of units (resulting in an increment of 45 for 3C/3D) is not recommended.

A/Prof Harrison’s figures # 1 to 6 in Appendix 3 indicate the use of an increment of 10 for the four pairs of units in mathematics provides results consistent with using an increment of 15 with a typical course with two pairs of units across stage 2 and stage 3.

What about the relativities between the results in Mathematics and the Mathematics: Specialist courses?

Over the years there have been significant concerns relating to students undertaking Discrete Mathematics and Applicable Mathematics and achieving significantly higher scaled marks in Discrete with relatively little work.

The new Mathematics course covers the content from the existing Discrete Mathematics through to Applicable Maths and the incentives will encourage students to undertake the highest unit pair they are capable of doing. The four exams in Mathematics will be scheduled at the same time so it will not be possible for a student to sit exams in two of the unit pairs of the Mathematics course in the same year.

It is expected that the Mathematics: Specialist course will attract very able students who would typically do well in their other courses and hence result in the Mathematics: Specialist scaled course average being significantly higher compared to the Mathematics scaled course average.

After the first year of the mathematics examinations, the Scaling Policy Committee will be reviewing the outcomes, as it will do for all courses.

Steve Hoath
TISC
STAGE 2 AND STAGE 3 COURSE RESULTS (Not Mathematics) FROM 2009 YEAR 12

STAGE 2

School Marks → Moderated School Marks → Standardised School Marks

Examination Marks → Standardised Examination Marks → Stage 2 Combined Marks

STAGE 3

School Marks → Moderated School Marks → Standardised School Marks

Examination Marks → Standardised Examination Marks → Stage 3 Combined Marks

Stage 3 Combined Marks → Stage 3 Combined Marks + 15

Stage 2 Combined Marks → Average Marks Scaling

COURSE Combined Marks → Scaled Scores

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Sum of best 4 → TEA → TER
Mathematics (MAT) Pathways

In a colour band, the lighter shade indicates Year 11, the darker, Year 12

### Mathematics (MAT) Pathways

<table>
<thead>
<tr>
<th>Entry points in terms of former subjects</th>
<th>Unit</th>
<th>Year</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ed. Support</td>
<td>PAMAT, PA</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PBMAT, PB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocational Mathematics</td>
<td>1AMAT, 1A, 1A</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Mathematics in Practice</td>
<td>1BMAT, 1B, 1B</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1CMAT, 1C, 1C</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1DMAT, 1D, 1D</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1EMAT, 1E, 1E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foundations of Mathematics</td>
<td>2AMAT, 2A, 2A</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2BMAT, public exam</td>
<td>2B</td>
<td></td>
</tr>
<tr>
<td>Foundations of Mathematics</td>
<td>2CMAT, 2C, 2C</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2DMAT, public exam</td>
<td>2D</td>
<td></td>
</tr>
<tr>
<td>Introductory Calculus</td>
<td>3AMAT, 3A, 3A</td>
<td>11</td>
<td>3AMAT</td>
</tr>
<tr>
<td>also Geometry and Trigonometry</td>
<td>3BMAT, public exam</td>
<td>3B</td>
<td>3BMAT</td>
</tr>
<tr>
<td></td>
<td>3CMAT,</td>
<td></td>
<td>3CMAT</td>
</tr>
<tr>
<td></td>
<td>3DMAT, public exam</td>
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<td>3DMAT</td>
</tr>
</tbody>
</table>

**Mathematics: Specialist (MAS) Pathways**

Mathematics: Specialist Standard Course (i.e. in conjunction with MAT units)

<table>
<thead>
<tr>
<th>Entry points in terms of former subjects</th>
<th>Specialist</th>
<th>Year</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introductory Calculus and Geometry and Trigonometry</td>
<td>3AMAS, 11</td>
<td></td>
<td>3AMAT</td>
</tr>
<tr>
<td></td>
<td>3BMAS, 11</td>
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<td>3BMAT</td>
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<tr>
<td></td>
<td>3CMAS, 12</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>3DMAS, 12</td>
<td></td>
<td>3DMAT</td>
</tr>
</tbody>
</table>

**HOWEVER:**
Mathematics: Specialist can be taken as a single course, stand-alone without MAT, depending on the student’s background and capability, but is only available following school counselling.
Setting the increment between the mathematics unit-pairs

The charts labelled #1, #2, ..., #6 show the effects of 3 possible choices for the increment that will be used to separate the unit-pairs within the mathematics course of study, for various distributions of students across these unit-pairs. They show quite conclusively that,

\begin{align*}
\text{to achieve parity with non-mathematics courses,} \\
\text{the increment between successive unit pairs in mathematics should be set at 10.}
\end{align*}

Each chart shows the proportions of students that can be expected to achieve various scaled scores in mathematics courses, for a particular distribution of students across unit-pairs. For the purpose of comparing mathematics with other courses of study, the mathematics cohorts have combined according to stages. Thus the 2A/B and 2C/D cohorts have been combined into a single stage 2 cohort, and similarly, the 3A/B and 3C/D cohorts have combined into a single stage 3 cohort.

The horizontal axis is the TER equivalent of the scaled course score. This is the TER that would be given to a student who obtained the same scaled score in 4 separate courses. For example, 80 is the TER equivalent of a scaled course score of 59, because a TEA of \( \frac{236594}{4} \) corresponds (by TISC 2007 data) to a TER of 80.

The vertical axis is the proportion of students obtaining a given TER equivalent score or better.

In chart #1, there are 8 curves. The top curve, which is labelled Math S3 I = 12, shows the achievement of mathematics students in a combined stage3 cohort if the mathematics increment is 12. It shows, for example, that about 70% of this cohort will achieve a scaled score which is TER-equivalent of 80, or better. The bottom curve, which is labelled Math S2 I = 12, shows the achievement of mathematics students in a combined stage2 cohort if the mathematics increment is 12. It shows, amongst other things, that about 23% of this cohort will achieve a scaled score which is TER-equivalent of 80, or better.

Not surprisingly, there are 2 ‘bands’ of curves. The top band corresponds to stage3 cohorts, and the bottom band corresponds to stage2 cohorts.

Within each band there are 4 curves. The two in the middle of each band are hard to distinguish, because their locations are almost identical. One of these corresponds to a mathematics cohort with the increment set at 10. The other is the curve that corresponds to a non-mathematics course of study that is typical in the sense that it is weighted neither up nor down by the Average Marks Scaling process. The same pattern is seen in the other five charts.

This shows that the results for mathematics with the increment set at 10 match remarkably well with the typical non-mathematics course, regardless of distribution of student numbers across the unit-pairs.

Charts #7 and #8 show the achievement rates for the individual unit-pairs within mathematics, for two possible student distributions, and with a mathematics increment of 10. They show, amongst other things, that there is a reasonably constant gap of about 20% in the achievement rates between successive unit pairs.

For comparative purposes, charts #9 and #10 include the actual achievement rates for the existing TEE mathematics subjects in 2007. They show that in terms of achievement rates

- Discrete Mathematics matches pretty closely with 2C/D
- Applicable Mathematics matches pretty closely with 3A/B
- Calculus lies about half-way between 3A/B and 3C/D.

It is important to bear in mind that these are not comparisons about the difficulty of the various unit-pairs in the new mathematics courses relative to the existing subjects. They are merely comparisons about the ‘reward structure’ for Tertiary Entrance.

K.J.Harrison
22 February 2008
Non-Maths distribution: Level 2: 50% Level 3: 50%
Mathematics distribution: 2A/B: 15% 2C/D: 35% 3A/B: 35% 3C/D: 15%

Non-Maths distribution: Level 2: 50% Level 3: 50%
Mathematics distribution: 2A/B: 20% 2C/D: 40% 3A/B: 30% 3C/D: 10%
Non-Maths distribution: Level 2: 60% Level 3: 40%  
Mathematics distribution: 2A/B: 20% 2C/D: 40% 3A/B: 30% 3C/D: 10%  

Non-Maths distribution: Level 2: 60% Level 3: 40%  
Mathematics distribution: 2A/B: 10% 2C/D: 50% 3A/B: 25% 3C/D: 15%
Non-Maths distribution: Level 2: 70% Level 3: 30%                                          #5
Mathematics distribution: 2A/B: 20%  2C/D: 50%  3A/B: 20% 3C/D: 10%

Non-Maths distribution: Level 2: 40% Level 3: 60%                                          #6
Mathematics distribution: 2A/B: 10%  2C/D: 30%  3A/B: 40% 3C/D: 20%
2A/B 15%  2C/D 35%  3A/B 35%  3C/D 15%

Scaled Score - TER equivalent

Achievement proportions (%)

2A/B 20%  2C/D: 40%  3A/B: 30%  3C/D: 10%

Scaled Score - TER equivalent

Achievement rate (%)

Units 2A/B  Units 2C/D  Units 3A/B  Units 3C/D
2A/B 15%  2C/D 35%  3A/B 35%  3C/D 15% + TEE subjects

2A/B 20%  2C/D 40%  3A/B 30%  3C/D 10% + existing TEE subjects
MATHEMATICS: SPECIALIST FROM 2010 YEAR 12

3A/3B
- Examination Marks
- Standardised Examination Marks
- School Marks → Moderated School Marks → Standardised Moderated School Marks

3C/3D
- Examination Marks
- Standardised Examination Marks
- School Marks → Moderated School Marks → Standardised Moderated School Marks

3A/3B Combined Marks
- COURSE Combined Marks
- Scaled Scores
- Sum of best 4
- TEA → TER

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